Thermal Buckling of Laminated Composite Shells

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Abstract

The linear buckling analysis of laminated composite cylindrical and conical shells under thermal load using the finite element method is reported here. Critical temperatures are presented for various cases of cross-ply and angle-ply laminated shells. The effects of radius/thickness ratio, number of layers, ratio of coefficients of thermal expansion, and the angle of fiber orientation have been studied. The results indicate that the buckling behavior of laminated shell under thermal load is different from that of mechanically loaded shell with respect to the angle of fiber orientation.

Content

HIGH-speed aerospace vehicles consisting of thin-shell elements are subjected to aerodynamic heating. This induces a temperature distribution over the surface and thermal gradient through the thickness of the shell. The compressive stresses, which in these circumstances develop, may cause buckling. Recently fiber-reinforced, laminated composites have begun to be used extensively in aerospace vehicle construction due to the high specific properties of the composites. In view of the above, the thermal buckling analysis of laminated composite shell assumes importance.

Thermal buckling of isotropic cylindrical and conical shells have been reviewed by Bushnell.[1] Chang and Card[2] investigated thermal buckling of stiffened, orthotropic, multilayered cylindrical shells. The governing equations obtained through the minimization of the total potential energy were solved by the finite difference technique with a few cases of practical problems. Nevertheless, this technique cannot be extended to other complex geometries and loading conditions. In this paper the Semiloof shell element formulated by Irons,[3] which was adapted by the authors for thermal stress analyses of laminated plates and shells,[4] is being extended to thermal buckling problems.

The derivation of governing equation is a standard procedure, which uses the principle of minimum total potential energy. The characteristic finite element equilibrium equation thus obtained is $[K_s]\{q\}=\{F\}$ where $[K_s]$ is the structural stiffness matrix, $\{q\}$ is the nodal displacement vector and $\{F\}$ is the consistent nodal load vector. In order to establish the critical buckling state corresponding to the neutral equilibrium condition, the second variation of the total potential must be equated to zero, which gives rise to the condition, $|[K_s]+\lambda[K_g]|=0$, where $[K_g]$ is the geometric stiffness matrix and λ is the eigenvalue. The computer program (COMSAP) developed based on this formulation can handle general temperature variations, lamination parameters, and various boundary conditions. The material properties considered in the analysis of laminated shells are $E_{11}/E_{tt}=10$, $G_{lt}/E_{tt}=0.5$, $\mu_{lt}=0.25$, $\alpha_t/\alpha_1=2$, $E_{11}=1.5\times10^4$ kg/cm^2, $\alpha_1=1\times10^{-6}$/°C (Material-*I*).

The shell is subjected to uniform temperature rise over the surface and through the thickness.

The critical temperature (T_{cr}) for the angle-ply cylindrical shells undergoing axisymmetric buckling modes are presented in Figs. 1–3. The T_{cr} reduces considerably with the fiber orientation angle with Θ varying from 0 deg to 90 deg for both symmeric and antisymmeric laminates. This is mainly due to the increase in the axial thermal stress resultant (N_{yy}) as Θ increases for the constant thermal load. For the assumed material properties, N_{yy} increases about 5 times; therefore the T_{cr} value (see Figs. 1 and 2) decreases to 1/5th as Θ changes from 0 deg to 90 deg. In the case of mechanical axial load, the buckling value is reported to vary symmetrically about 45 deg in Ref. 5, and the applied load does not change with the angle of fiber orientation. Also in the case of square laminated plates subjected to thermal loads, T_{cr} is symmetrically about 45 deg when all the edges have same boundary condition, as reported in Ref. 6. This is due to the biaxial nature of the stresses induced in the plate by the thermal load. In cylindrical shells the hoop stress is developed only at the boundary due to constraints, and the axial stress resultant alone controls the buckling. The effect of ratio of coefficients of thermal expansion (α_t/α_1) is shown in Fig. 3. It may be seen that for smaller values of the ratio, the fiber orientation angle has a pro-

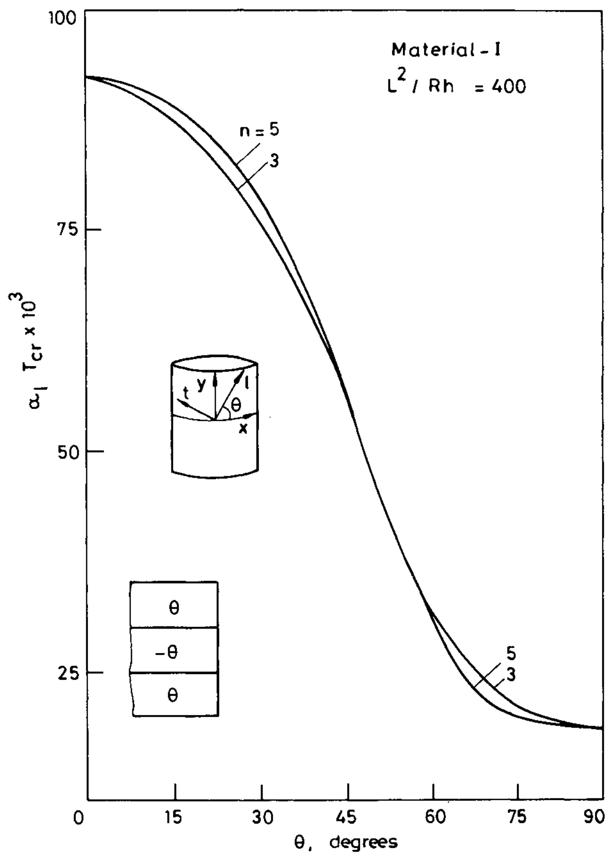

Fig. 1 Critical temperature of symmetric, angle-ply cylindrical shells under constant temperature.

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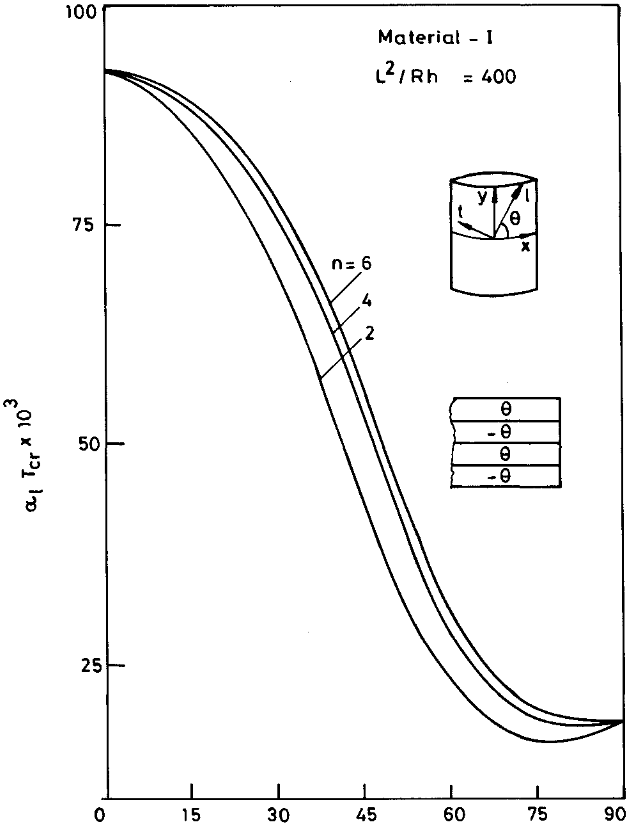

Fig. 2 Critical temperature for antisymmetric, angle-ply cylindrical shells under constant temperature.

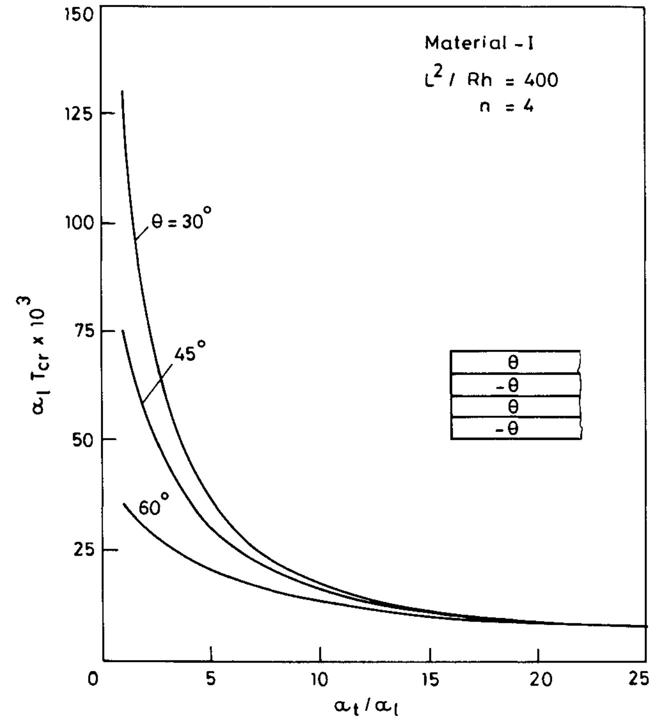

Fig. 3 Variation of critical temperature with the coefficient of thermal expansion ratio.

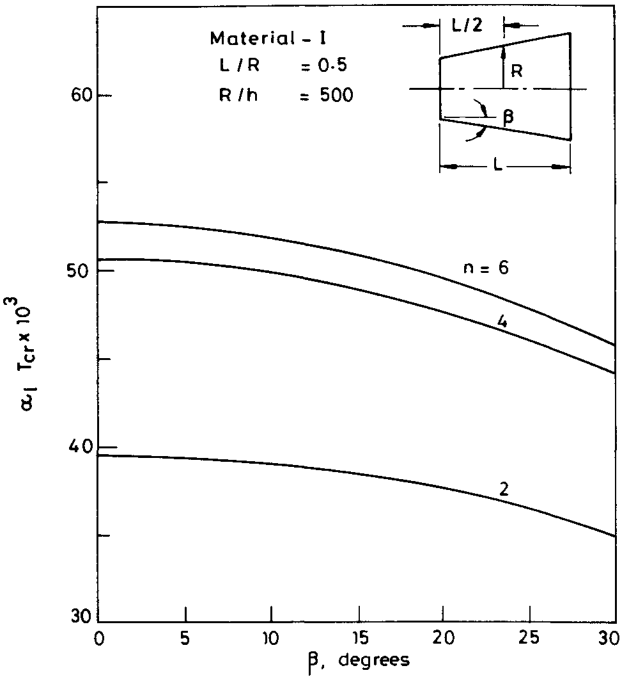

Fig. 4 Critical temperature for laminaed, cross-ply conical shells.

nounced effect. That is, the lower angle of fiber orientation gives rise to a higher T_{cr}, and considerable reduction is seen as the angle of orientation increases. For higher values of ratio ($\alpha_t/\alpha_l > 10$) the effect of fiber orientation diminishes.

The critical temperature for conical shells with respect to the semivertex angle β is shown in Fig. 4. The variation is observed to be small for the range of angles considered. For the limiting values of $\beta = 0$, the value corresponds to that of cylindrical shell. Also shown in the figure is the effect of the number of layers on the critical temperature. The difference between the critical temperatures of 2-layer and 4-layer laminates is about 25%.

References

[1]Bushnell, D., "Computerized Buckling Analysis of Shells," Martinus Nijhoff Publishers, Dordrecht, the Netherlands, 1985, pp. 74–83.

[2]Chang, L. K. and Card, M. F., "Thermal Buckling Analysis for Stiffened Orthotropic Cylindrical Shells," NASA TND6332, April 1971.

[3]Irons, B. M., "The Semiloof Shell Elements," *Proceedings of the Conference on Finite Elements for Thin Shells,* edited by D. G. Ashwell and R. H. Gallagher, Wiley, London, 1976, pp. 197–222.

[4]Thangaratnam, R., Palaninathan, R. and Ramachandran, J., "Thermal Stress Analysis of Laminated Composite Plates and Shells," *Computers and Structures,* Vol. 30, 1988, pp. 1403–1411.

[5]Uemura, M. and Kasuya, A., "Coupling Effect of Axial Compressive Buckling of Laminated Composite Cylindrical Shells," *Progress in Science and Engineering of Composites,* edited by T. Hayashi, K. Kawata, and S. Umakawa, S., Proceedings of the Fourth International Conference on Composite Materials, Tokyo, 1982, pp. 583–590.

[6]Thangaratnam, R., Palaninathan, and Ramachandran, J., "Thermal Buckling of Composite Laminated Plates," *Computers and Structures,* Vol. 32, 1989, 1117–1124.